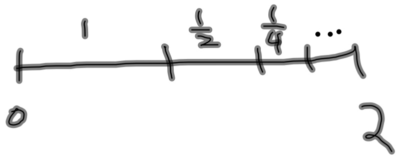
9.5 Infinite Series, Power Series

a = first term

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$$

infinite geometric series

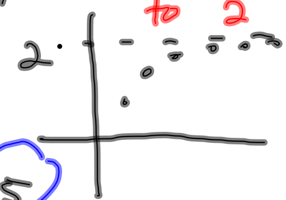


multiply by $\frac{1}{2}$

common ratio (r)

Feb 10-11:56 AM

sequence of partial sums – converges to 2



n 1^{st} partial sum = S_n (1)

2^{nd} partial sum = $1 + \frac{1}{2}$ = 1.5

3^{rd} partial sum = $1 + \frac{1}{2} + \frac{1}{4}$ = 1.75

$$u1(n) = \sum_{k=1}^{n} \frac{1}{2^{k-1}} \qquad 2$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 2$$

Feb 10-12:00 PM

geometric series

$$a + ar + ar^2 + ar^3 \ldots = \sum_{n=1}^{\infty} ar^{n-1}$$

$$= \frac{a}{1-r} \quad \text{if } |r| < 1$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = \sum_{n=1}^{\infty} 1\left(\frac{1}{2}\right)^{n-1}$$

$$= \frac{1}{1-\frac{1}{2}} = \frac{1}{1/2} = 2$$

Feb 10-12:10 PM

$$2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots \stackrel{?}{=} \frac{2}{1 - -\frac{2}{3}} = \frac{2}{\frac{5}{3}} = \frac{6}{5}$$

$r = -\frac{2}{3}$ $a = 2$

seq of partial sums

$$\sum_{n=1}^{\infty} 2\left(-\frac{2}{3}\right)^{n-1} ; \quad u1(n) = \sum_{k=1}^{n} 2\left(-\frac{2}{3}\right)^{k-1}$$

$$\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^{n-1} \stackrel{?}{=} \infty \text{ diverges}$$

since $|r| \geq 1$

$r = \frac{5}{4}$

Feb 10-12:15 PM

a new way to define a function

$$1 + x + x^2 + x^3 + x^4 + \dots = \frac{1}{1-x}$$

power series, also geo. $a=1$

$r = x$

if $|r| < 1$

$-1 < x < 1$ — $|x| < 1$

$\frac{a}{1-r}$

interval of convergence

Feb 10-12:23 PM

what power series?

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 \dots$$

$|-x| < 1$

$a = 1$

$r = -x$

integrate both sides

$$\int \frac{1}{1+x}\,dx = \int 1 - x + x^2 - x^3 + x^4 \dots$$

$$\ln|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \dots$$

let $x=1$ $\quad \ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots$

Feb 10-12:30 PM

$$(1-x)^{-1} = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 \ldots$$

der. of both sides

$$\frac{+1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + 4x^3 \ldots$$

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find power series for $\tan^{-1} x$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 \ldots$$

$a = 1 \quad r = -x^2$

$$\tan^{-1} x = \int 1 - x^2 + x^4 - x^6$$

Feb 10-12:37 PM